JEE Advanced 2026

Sample Paper - 5 (Paper-1)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +4 marks for the correct response and -1 for the incorrect response.

Physics

1. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then: **[3]**

a) $T_A = T_B$

b) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$

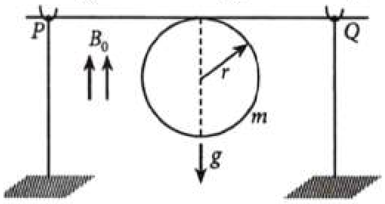
c) $T_A > T_B$ (if $m_A > m_B$)

d) $T_A > T_B$ (if $r_A > r_B$)

2. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a [3]

a) convergent lens of focal length 3.0 R
b) convergent lens of focal length 3.5 R
c) divergent lens of focal length 3.0 R
d) divergent lens of focal length 3.5 R

3. A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and it is in a uniform vertical magnetic field B_0, as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity g, on two conducting supports at P and Q. When a current I is passed through the loop, the loop turns about the line PQ by an angle θ given by [3]



a) $\tan\theta = mg/(\pi r I B_0)$
b) $\tan\theta = \pi r I B_0/(mg)$
c) $\tan\theta = 2\pi r I B_0/(mg)$
d) $\tan\theta = \pi r I B_0/(2mg)$

4. Two solid cylinders P and Q of the same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? [3]

a) Both cylinders reach the ground with same translational kinetic energy
b) Both cylinders P and Q reach the ground at the same time
c) Cylinder P has larger linear acceleration than cylinder Q
d) Cylinder Q reaches the ground with larger angular speed

5. The electric field associated with an electromagnetic wave propagating in a dielectric medium is given $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right] Vm^{-1}$. [4]
Which of the following option(s) is (are) correct?
[Given: The speed of light in vacuum, $c = 3 \times 10^8$ m s^{-1}]

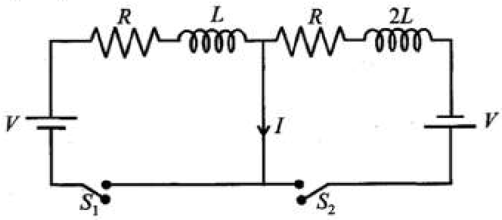
a) $B_x = -2 \times 10^{-7} \sin\left[2\pi\left(5 \times 10^{14}t - \frac{10^7}{3}z\right)\right] \text{Wbm}^{-2}$

b) The refractive index of the medium is 2.

c) The wave is polarized in the xy-plane with polarization angle 30^o with respect to the x-axis

d) $B_y = 2 \times 10^{-7} \sin\left[2\pi\left(5 \times 10^{14}t - \frac{10^7}{3}z\right)\right] \text{Wbm}^{-2}$.

6. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statements is (are) true? **[4]**



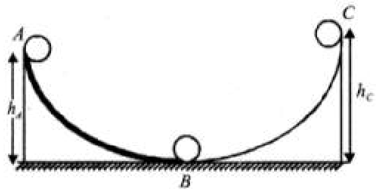
a) $\tau = \frac{L}{R}\ln 2$

b) $\tau = \frac{2L}{R}\ln 2$

c) $I_{max} = \frac{V}{4R}$

d) $I_{max} = \frac{V}{2R}$

7. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_c are kinetic energies of the ball at A, B and C, respectively. Then **[4]**



a) $h_A > h_C; K_B > K_C$

b) $h_A = h_C; K_B = K_C$

c) $h_A > h_C; K_C > K_A$

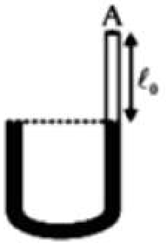
d) $h_A < h_C; K_B > K_C$

8. An optical bench has a 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at the 75 cm mark of the scale and the object pin is kept at the 45 cm mark. The image of the object pin on the other side of the lens overlaps with the image pin that is kept at **[4]**

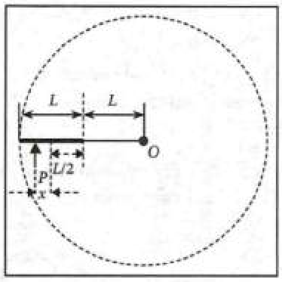
the 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is ________.

9. A steady current I goes through a wire loop PQR having shape of a right-angle triangle with PQ = 3x, PR= 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$ flnd the value of k. **[4]**

10. An experiment is carried out under pressure $P_0 = 100\ cm$ of Hg and consists of a U tube of uniform cross section is in vertical position as shown. Now end A of tube is closed and gas in the tube is heated so that gas expands and mercury spills out. During the process, it is seen that the pressure of enclosed gas is directly proportional to the volume of gas. Find ℓ_0 (in m). **[4]**



11. A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance x = L/n from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is ________. **[4]**



12. A liquid with coefficient of volume expansion $10^{-3}/^\circ C$ fills a vessel of internal volume 1 liter up to the brim. The vessel has a coefficient of linear expansion of $5 \times 10^{-5}/^\circ C$. The original density of liquid is $1 gm/cc$. The vessel is heated by $20^\circ C$ and the liquid spills over. If mass of the liquid that spills over is $(25 - x) gm$, find the value of x - (Round off to the nearest integer). **[4]**

13. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction ${}^{16}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{1}_{1}\mathrm{H} + {}^{19}_{8}\mathrm{O}$ in a laboratory frame in n (in MeV). Assume that ${}^{16}_{7}\mathrm{N}$ is at rest in the laboratory frame. The masses of ${}^{16}_{7}\mathrm{N}$, ${}^{4}_{2}\mathrm{He}$, ${}^{1}_{1}\mathrm{H}$ and ${}^{19}_{8}\mathrm{O}$ can **[4]**

be taken to be 16.006u, 4.003u, 1.008u and 19.003u, respectively, where $1u = 930 \mathrm{MeV c^{-2}}$. The value of n is

14. Match List I with List II: **[4]**

List I	List II
(A) 3 Translational degrees of freedom	(I) Monoatomic gases
(B) 3 Translational, 2 rotational degrees of freedoms	(II) Polyatomic gases
(C) 3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III) Rigid diatomic gases
(D) 3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV) Nonrigid diatomic gases

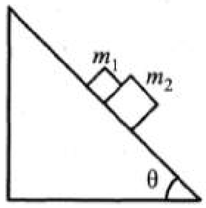
a) (A) - (IV), (B) - (III), (C) - (IV), (D) - (I)

b) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)

c) (A) - (I), (B) - (IV), (C) - (III), (D) - (II)

d) (A) - (IV), (B) - (II), (C) - (I), (D) - (IIII)

15. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List-I. The coefficient of friction between the block m_1 and plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List-II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by g. **[4]**
[Useful information: $\tan(5.5^\circ) \approx 0.1; \tan(11.5^\circ) \approx 0.2; \tan(16.5^\circ) \approx 0.3$



List - I	List - II
(P) $\theta = 5^\circ$	(i) $m_2 g \sin\theta$
(Q) $\theta = 10^\circ$	(ii) $(m_1 + m_2) g \sin\theta$
(R) $\theta = 15^\circ$	(iii) $\mu m_2 g \cos\theta$
(S) $\theta = 20^\circ$	(iv) $\mu (m_1 + m_2) g \cos\theta$

a) (P) - (ii), (Q) - (ii), (R) - (ii), (S) -(iii)

b) (P) - (ii), (Q) - (ii), (R) - (iii), (S) - (iii)

c) (P) - (i), (Q) - (i), (R) - (i), (S) - (iii)

d) (P) - (ii), (Q) - (ii), (R) - (ii), (S) - (iv)

16. Four physical quantities are listed in Column I. Their values are listed in Column II in a random order: **[4]**

Column I	**Column II**
(a) Thermal energy of air molecules at room temp	(e) 0.02 eV
(b) Binding energy of heavy nuclei per nucleon	(f) 2eV
(c) X-ray photon energy	(g) 1 keV
(d) Photon energy of visible light	h) 7 MeV

The correct matching of Columns I and II is given by

a) a - f, b - h, c - e, d - g

b) a - e, b - h, c - g, d - f

c) a - f, b - e, c - g, d - h

d) a - e, b - g, c - f, d - h

Chemistry

17. The temporary hardness of water due to calcium bicarbonate can be removed by adding- **[3]**

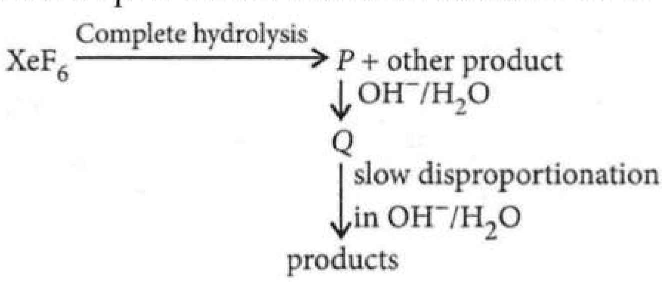
a) HCl

b) $Ca(OH)_2$

c) $CaCO_3$

d) $CaCl_2$

18. Under ambient conditions, the total number of gases released as products in the final step of the reaction scheme shown below is **[3]**



a) 0

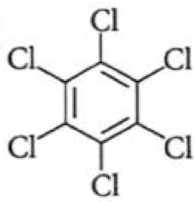
b) 3

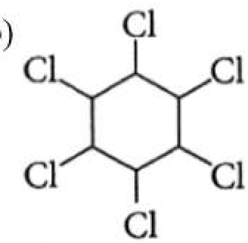
c) 1

d) 2

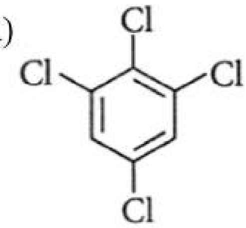
19. In the following reaction sequence, the major product Q is **[3]**

$$L\text{-Glucose} \xrightarrow[\text{(ii) } Cr_2O_3,\ 775\ K,\ 10\text{–}20\ atm]{\text{(i) HI, }\Delta} P \xrightarrow[UV]{Cl_2\ \text{(excess)}} Q$$

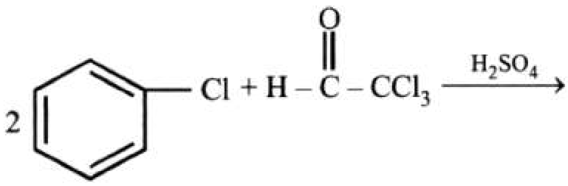
a) 

b) 

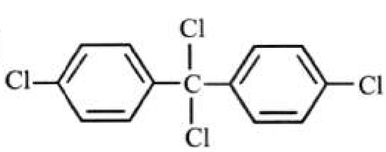
c) 

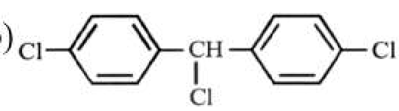
d) 

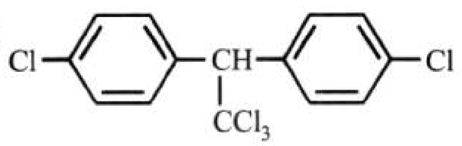
20. Chlorobenzene reacts with trichloro acetaldehyde in the presence of H_2SO_4. **[3]**

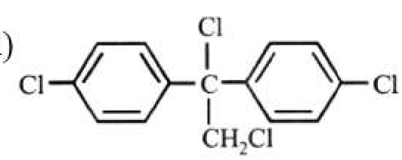


The major product formed is:

a) 

b) 

c) 

d) 

21. For the reaction **[4]**

$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$

The correct statement(s) in the balanced equation is/are

a) H_2O is one of the products

b) Sulphur is reduced

c) Stoichiometric coefficient of HSO_4^- is 6

d) Iodide is oxidized

22. The products of reaction of alcoholic silver nitrite with ethyl bromide are **[4]**

a) ethyl alcohol

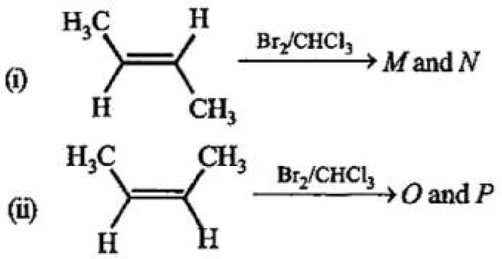
b) ethene

c) nitroethane

d) ethyl nitrite

e) ethane

23. The correct statement(s) for the following addition reactions is (are) **[4]**



a) (M and O) and (N and P) are two pairs of diastereomers

b) O and P are identical molecules

c) Bromination proceeds through trans-addition in both the reactions

d) (M and O) and (N and P) are two pairs of enantiomers

24. The 1st, 2nd, and the 3rd ionization enthalpies, I_1, I_2, and I_3, of four atoms with atomic numbers n, n + 1, n + 2, and n + 3, where n < 10, are tabulated below. What is the value of n? **[4]**

Atomic number	**Ionization Enthalpy (kJ/mol)**		
	I_1	I_2	I_3
n	1681	3374	6050
n + 1	2081	3952	6122
n + 2	496	4562	6910
n + 3	738	1451	7733

25. Among the complex ions, $[Co(NH_2CH_2CH_2\text{-}NH_2)_2\,Cl_2]^+$, $[CrCl_2\,(C_2O_4)_2]^{3-}$, $[Fe(H_2O)_4(OH)_2]^+$, $[Fe(NH_3)_2(CN)_4]^-$, $Co(NH_2\text{-}CH_2\text{-}CH_2\text{-}NH_2)_2(NH_3)Cl]^{2+}$ and $[Co(NH_3)_4(H_2O)Cl]^{2+}$ the number of complex ion(s) that show(s) cis-trans isomerism is **[4]**

26. To form a complete monolayer of acetic acid on 1 g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies $P \times 10^{-23}\ m^2$ surface area on charcoal, the value **[4]**

of P is ________.

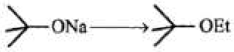
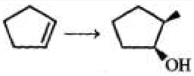
[Use given data: Surface area of charcoal $= 1.5 \times 10^2\ m^2\ g^{-1}$; Avogadro's number $(N_A) = 6.0 \times 10^{23}\ mol^{-1}$]

27. Calculate the wave number for the shortest wavelength transition in the Balmer series of atomic hydrogen. **[4]**

28. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is **[4]**

29. The rate of a first-order reaction is 0.04 mol $litre^{-1}\ s^{-1}$ at 10 minutes and 0.03 mol $litre^{-1}\ s^{-1}$ at 20 minutes after initiation. Find the half-life of the reaction. **[4]**

30. Match the chemical conversions in **List-I** with the appropriate reagents in **List-II** and select the correct answer using the code given below the lists : **[4]**

(List - I)	**(List-II)**
P 	1. (i) Hg $(OAc)_2$; (ii) $NaBH_4$
Q 	2. NaOEt
R 	3. Et-Br
S 	4.(i)BH_3; (ii) $\frac{H_2O_2}{NaOH}$

a) (P) - (2), (Q) - (3), (R)- (1), (S) - (4)

b) (P) - (2), (Q) - (3), (R)- (4), (S) - (1)

c) (P) - (3), (Q) - (1), (R)- (2), (S) - (4)

d) (P) - (3), (Q) - (2), (R)- (1), (S) - (4)

31. Match List I with List II **[4]**

List-I **Name of reaction**	**List-II** **Reagent used**
(A) Hell-Volhard-Zelinsky reaction	(I) NaOH + I_2
(B) Iodoform reaction	(II) (i) CrO_2Cl_2,CS_2(ii)H_2O
(C) Etard reaction	(III) (i) Br_2/red phosphorus (ii) H_2O
(D) Gatterman-Koch reaction	(IV) CO, HCI, anhyd, $AlCl_3$

a) A - III, B - I, C - IV, D - II

b) A - III, B - II, C - I, D - IV

c) A - I, B - II, C - III, D - IV

d) A - III, B - I, C - II, D - IV

32. The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option. **[4]**

List-I	**List-II**
(P) Etard reaction	(1) Acetophenone $\xrightarrow[HCl]{Zn-Hg}$
(Q) Gattermann reaction	(2) Toluene $\xrightarrow[\text{(ii)}SOCl_2]{\text{(i)}KMnO_4, KOH, \Delta}$
(R) Gattermann-Koch reaction	(3) Benzene $\xrightarrow[\text{anhyd.} AlCl_3]{CH_3Cl}$
(S) Rosenmund reduction	(4) Aniline $\xrightarrow[273-278K]{NaNO_2/HCl}$
	(5) Phenol $\xrightarrow{Zn,\ \Delta}$

a) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$

b) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 5; S \rightarrow 2$

c) $P \rightarrow 2; Q \rightarrow 4; R \rightarrow 1; S \rightarrow 3$

d) $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 5; S \rightarrow 2$

Maths

33. Number of positive integers n less than 15, for which n! + (n+1)! + (n+2)! is an integral
multiple of 49, is **[3]**

a) 6

b) 4

c) 3

d) 5

34. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that: **[3]**

a) PX = -X

b) PX = X

c) PX = 2X

d) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

35. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy = 0$ (where, $pq \neq 0$) are bisected by the x - axis, then **[3]**

a) $p^2 < 8q^2$
b) $p^2 = q^2$
c) $p^2 > 8q^2$
d) $p^2 = 8q^2$

36. Let $f(x) = (x + 1)^2 - 1, x \geq -1$, Then the set $\{x : f(x) = f^{-1}(x)\}$ is **[3]**

a) $\{0, -1\}$
b) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$
c) empty
d) $\{0, 1, -1\}$

37. Let $f : [a, b] \rightarrow [1, \infty)$ be a continuous function and let $g : R \rightarrow R$ be defined as **[4]**

$$g(x) = \begin{cases} 0, & \text{if } x < a, \\ \int_a^x f(t)dt, & \text{if } a \leq x \leq b; \\ \int_a^b f(t)dt, & \text{if } x > b. \end{cases} \quad \text{then}$$

a) g(x) is continuous and differentiable at either (a) or (b) but not both
b) g(x) is continuous but not differentiable at b
c) g(x) is differentiable on R
d) g(x) is continuous but not differentiable at a

38. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) **[4]**

a) 1
b) 2
c) 3
d) 4

39. Let $a, b \in R$ and $a^2 + b^2 \neq 0$. **[4]**
Suppose $S = \left\{z \in C : Z = \frac{1}{a+ibt}, \in R^+, t \neq 0\right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on

a) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$
b) the y-axis for $a = 0, b \neq 0$
c) the x-axis for $a \neq 0, b = 0$
d) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq$

0

40. The maximum value of the function **[4]**
$f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set
$A = \{x | x^2 + 20 \leq 9x\}$ is

41. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill [in meters] is ________. **[4]**

42. Consider the lines L_1 and L_2 defined by **[4]**
$L_1: x\sqrt{2} + y - 1 = 0$ and $L_2: x\sqrt{2} - y + 1 = 0$
For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ_2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.
Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the **square** of the distance between R' and S'. The value of λ^2 is ________.

43. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, ..., 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40. The value of $\frac{125}{4}$ p_2 is ________. **[4]**

44. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18, then the value of the determinant of A is ________. **[4]**

45. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of g'(1) is **[4]**

46. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5. **[4]**
   i. Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.
   ii. Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.
   iii. Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

iv. Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I	**LIST-II**
(P) The value of α_1 is	(1) 136
(Q) The value of α_2 is	(2) 189
(R) The value of α_3 is	(3) 192
(S) The value of α_4 is	(4) 200
	(5) 381
	(6) 461

a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 1$ b) $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 2; S \rightarrow 1$

c) $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 3$ d) $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 5; S \rightarrow 2$

47. Consider the given data with frequency distribution **[4]**

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in **List-I** to the correct entries in **List-II**.

List-I	**List-II**
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7
	(5) 2.4

a) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (5) b) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (5)

c) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1) d) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (5), (S) $\rightarrow$ (5)

48. Match List I with List II and select the correct answer using the code given below the lists: **[4]**

List-I	List-II
(P) Let $y(x)=\cos\left(3\cos^{-1}x\right), x\in[-1,1], x\neq\pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{\left(x^2-1\right)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals	(1) 1
(Q) Let $A_1, A_2, \ldots, A_n$ ($n>2$) be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, k = 1, 2, ...,n. If $\left\lvert\sum_{k=1}^{n-1}\left(\overrightarrow{a_k}\times\vec{a}_{k+1}\right)\right\rvert=\left\lvert\sum_{k=1}^{n-1}\left(\overrightarrow{a_k}\cdot\overrightarrow{a_{k+1}}\right)\right\rvert$, then the minimum value of n is	(2) 2
(R) If the normal from the point $P(h,1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line x + y = 8, then the value of h is	(3) 8
(S) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is	(4) 9

a) $P\rightarrow 2, Q\rightarrow 4, R\rightarrow 3, S\rightarrow 1$ b) $P\rightarrow 4, Q\rightarrow 3, R\rightarrow 1, S\rightarrow 2$

c) $P\rightarrow 4, Q\rightarrow 3, R\rightarrow 2, S\rightarrow 1$ d) $P\rightarrow 2, Q\rightarrow 4, R\rightarrow 1, S\rightarrow 3$

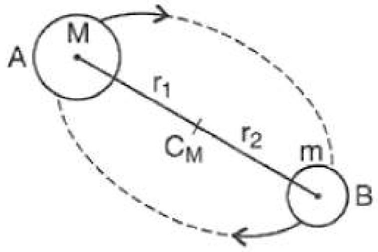
Solution

Physics

1. **(a)** $T_A = T_B$

Explanation:

If two binary stars are rotating about their common centre of mass, they have to be on the same line all the time as otherwise, the centre of mass will be changing. Their angular velocities have to be the same although in the same time the smaller mass will describe a bigger circle.



But as $\omega = \frac{2\pi}{T}$

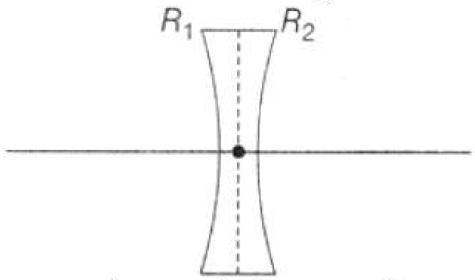
So, $T_A = T_B$

2.

(b) convergent lens of focal length 3.5 R

Explanation:

$R_1 = -R$, $R_2 = +R$, $\mu_g = 1.5$ and $\mu_m = 1.75$



$\therefore \frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Substituting the values, we have

$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right) = \frac{1}{3.5R}$

$\therefore f = +3.5R$

Therefore, in the medium, it will behave like a convergent lens of focal length 3.52. It can be understood as, $\mu_m > \mu_g$, the lens will change its behavior.

3.

(b) $\tan\theta = \pi r I B_0/(mg)$

Explanation:

$\tau_0 = MB_0 \cos\theta \ldots$ (i)

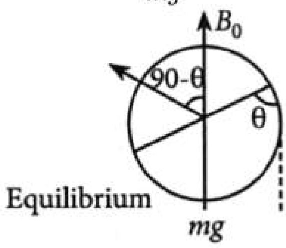
$\tau = mgr \sin\theta$ ...(ii)

Equate, eq (i) and (ii),

$MB_0 \cos\theta = mgr \sin\theta$

$I.\pi r^2 B_0 \cos\theta = mgr \sin\theta \left(\because M = I\pi r^2\right)$

$\tan\theta = \frac{I\pi r B_0}{mg}$



4.

(d) Cylinder Q reaches the ground with larger angular speed

Explanation:

$I_P > I_Q$

In case of pure rolling, $a = \frac{g \sin\theta}{1 + I/mR^2}$

$a_Q > a_P$ as its moment of inertia is less. Therefore, Q reaches first with more linear speed and more translational kinetic energy.

Further, $\omega = \frac{v}{R}$ or $\omega \propto v$

$\therefore \quad \omega_Q > \omega_P$ as $\nu_P > v_Q$

5. **(a)** $B_x = -2 \times 10^{-7} \sin\left[2\pi\left(5 \times 10^{14} t - \frac{10^7}{3} z\right)\right] \mathrm{Wbm}^{-2}$.

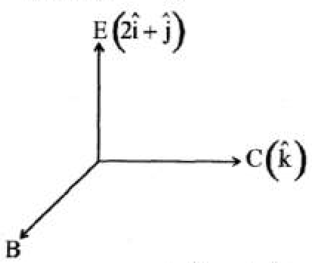
(b) The refractive index of the medium is 2.

Explanation: Speed of light in medium, $C_{medium} = \frac{\omega}{K} = \frac{5 \times 10^{14}}{\frac{10^7}{3}}$

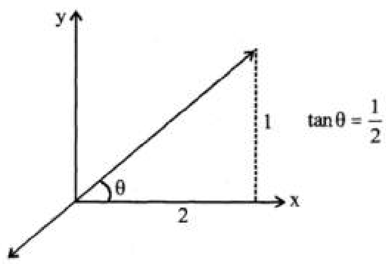
$= 1.5 \times 10^8$ m/s $\therefore \mu = \frac{C_{air}}{C_{medium}} = 2$

Also,

$C_{medium} = \frac{E}{B} \Rightarrow B = \frac{E}{C_m} = \frac{30\sqrt{5}}{1.5 \times 10^8} = 2\sqrt{5} \times 10^{-7}$



$B_{direction} = \vec{V} \times \vec{E} \equiv \hat{k} \times (2\hat{i} + \hat{j}) = \frac{2\hat{j} - \hat{i}}{\sqrt{5}}$



$\therefore B_x = 2 \times 10^{-7}(-\hat{i} + 2\hat{j}) \sin\left[27\left(5 \times 10^{17}t - \frac{10^7}{3}z\right)\right]$

6. **(b)** $\tau = \frac{2L}{R}\ln 2$

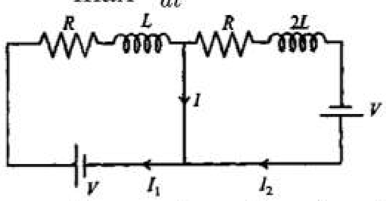
(c) $I_{max} = \frac{V}{4R}$

Explanation: Here $I + I_2 = I_1 \therefore I = I_1 - I_2$

$\therefore I = \frac{V}{R}\left[1 - e^{\frac{-Rt}{2L}}\right] - \frac{V}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$

$\Rightarrow I = \frac{V}{R}\left[e^{\frac{-Rt}{L}} - e^{\frac{-Rt}{2L}}\right]$ ...(i)

For I_{max}, $\frac{dI}{dt} = 0$



$\therefore \frac{V}{R}\left[\frac{-R}{L}e^{\frac{-Rt}{L}} - \left(\frac{-R}{2L}\right)e^{\frac{-Rt}{2L}}\right] = 0$

$\therefore e^{\frac{-Rt}{2L}} = \frac{1}{2} \Rightarrow \left(\frac{R}{2L}\right)t = \ln 2 \therefore t = \frac{2L}{R}\ln 2$

This is the time when I is maximum

Putting this value of time in eq.(i)

Further $I_{max} = \frac{V}{R}\left[e^{\frac{-R}{L}\left(\frac{2L}{R}\ell n2\right)} - e^{\frac{-R}{2L}\left(\frac{2L}{R}\ell n2\right)}\right]$

$\Rightarrow I_{max} = \frac{V}{R}\left[\frac{1}{4} - \frac{1}{2}\right] = \frac{V}{4R}$

7. **(a)** $h_A > h_C; K_B > K_C$

(c) $h_A > h_C; K_C > K_A$

(d) $h_A < h_C; K_B > K_C$

Explanation: From figure given in question,

Potential energy of the ball at point A = mgh_A

Potential energy of the ball at point B = 0

Potential energy of the ball at point C = mgh_c

Total energy at point A, $E_A = K_A + mgh_A$

Total energy at point B, $E_B = K_B$

Total energy at point C, $E_c = K_C + mgh_c$

As body rolls between A and B and between B and C there is no friction. So energy should be conserved here

By law of conservation of energy.

$E_A = E_B = E_C$

As, $E_A = E_C$

$K_A + mgh_A = K_C + mgh_C$

So, If $h_A > h_C \Rightarrow K_A < K_C$. So option (b) is correct

If $h_A < h_C \Rightarrow K_A > K_C$

Doesn't matter if $h_A > h_C$ or $h_A < h_C$ we will always have

$K_B > K_C$ because $E_A = E_B = E_C$.

8. 1.39

Explanation:

$u \pm \Delta u = (75 - 45) \pm \left(\frac{1}{4} + \frac{1}{4}\right) = (30 \pm 0.5)$ cm

$v \pm \Delta v = (135 - 75) \pm \left(\frac{1}{4} + \frac{1}{4}\right) = (60 \pm 0.5)$ cm

We know that

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore + \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$ ...(i)

Now $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} - \frac{1}{-30} = \frac{1}{f}$

$\Rightarrow \frac{1}{f} = \frac{1}{20} \therefore f = 20$ cm

Substituting the values in eqn. (i)

$\frac{0.5}{(60)^2} + \frac{0.5}{(30)^2} = \frac{\Delta f}{(20)^2}$

$\therefore \Delta f = 0.277$

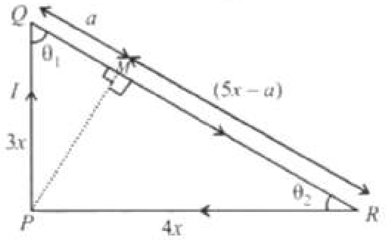
Hence percentage error in the measurement of focal length

$\frac{\Delta f}{f} \times 100 = \frac{0.277}{20} \times 100 = 1.388\% = 1.39\%$

9. 7

Explanation:

The magnetic field B due to wires PR and PQ = 0. Only wire QR will produce magnetic field at P. From point P, PM $\perp$ QR



Magnetic field at 'P' due to wire RQ

$B = \frac{\mu_0}{4\pi} \frac{I}{PM} (\cos\theta_1 + \cos\theta_2)$ ... (i)

In $\triangle$PQM $9x^2 = PM^2 + a^2$ ... (ii)

In $\triangle$PRM,

$16x^2 = PM^2 + (5x - a)^2$ ... (iii)

$\Rightarrow 7x^2 = 25x^2 - 10xa$

$\Rightarrow 10xa = 18x^2$

$\Rightarrow$ a = 1.8x ... (iv)

From eq. (ii) & (iv),

$9x^2 = PM^2 + (1.8x)^2$

$\therefore$ PM = $\sqrt{9x^2 - 3.24x^2} = \sqrt{5.76x^2}$ = 2.4x ... (v)

Also, $\cos\theta_1 = \frac{a}{3x} = \frac{1.8x}{3x}$ = 0.6 ... (vi)

$\cos\theta_2 = \frac{5x-a}{4x} = \frac{5x-1.8x}{4x} = \frac{3.2}{4}$ = 0.8 ... (vii)

Therefore, from eq. (i), (v), (vi) and (vii),

$B = \frac{\mu_0}{4\pi} \times \frac{I}{2.4x}[0.6 + 0.8]$

$= \frac{\mu_0}{4\pi} \times \frac{I}{2.4x} \times 1.4 = 7\left[\frac{\mu_0 I}{48\pi x}\right]$

Comparing it with $B = k\left[\frac{\mu_0 I}{48\pi x}\right]$

we get, k = 7

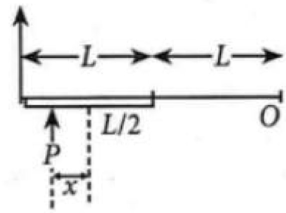
10. 1.0

Explanation:

1

11. 18

Explanation:



Moment of inertia of rod about centre

$I = \frac{mL^2}{12} + m\left(L + \frac{L}{2}\right)^2 = \frac{7mL^2}{3}$

As rod is rotating about O,

Angular impulse $= P\left(x + \frac{3L}{2}\right) = I\omega_0 = \frac{7mL^2\omega_0}{3}$

Linear impulse, $P = mv_c$ and $v_c = \frac{3L}{2}\omega_0$

$\therefore P = m \times \frac{3L}{2}\omega_0$

Substituting eq. (ii) in eq. (i)

$x + \frac{3L}{2} = \frac{14}{9}L; x = \frac{L}{18}$

12. 8.0

Explanation:

8

13. 2.33

Explanation:

We have

$Q = \Delta mc^2$

$= (m_N + m_{He} - m_H - m_O) \times 930MeV$

$= (16.006 + 4.003 - 1.008 - 19.003) \times 930$ MeV

$= -1.86$ MeV $= 1.86$ MeV energy absorbed

This Q is equal to maximum loss in kinetic energy of α-particle.

Considering collision as inelastic, we get maximum loss in

$$\text{K.E.} = \frac{1}{2} \times \left(\frac{4\text{ m} \times 16\text{ m}}{4\text{ m} + 16\text{ m}}\right) \times \text{v}^2$$

$$\Rightarrow Q = \left(\frac{1}{2} \times 4m \times v^2\right) \times \frac{16m}{20m} \Rightarrow Q = (\text{ K.E. })_{\min} \times \frac{4}{5}$$

$$\Rightarrow (\text{ K.E. })_{\min} = \frac{5}{4}\text{Q} = \left(\frac{5}{4} \times 1.86\right)\text{ MeV} = 2.325\text{MeV}$$

$\therefore \text{n} = 2.33$

14.

(b) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)

Explanation:

Types of gases	No. of degrees of freedom
Monoatomic gas	3T
Diatomic + rigid	3T + 2R
Diatomic + non-rigid	3T + 2R + 1V
Polyatomic	3T + 3R + More than 1V

T = Translational degree of freedom

R = Rotational degree of freedom

V = Vibrational degree of freedom

15.

(b) (P) - (ii), (Q) - (ii), (R) - (iii), (S) - (iii)

Explanation:

(P) - (ii), (Q) - (ii), (R) - (iii), (S) - (iii)

16.

(b) a - e, b - h, c - g, d - f

Explanation:

At room temperature, thermal energy of air molecule $= 0.02$eV

photon energy of visible light $(\lambda = 4000\overset{o}{A}$ to $700\overset{o}{A}) = 2$eV.

Chemistry

17.

(b) $Ca(OH)_2$

Explanation:

Temporary hardness of water is due to the presence of bicarbonates of Ca and Mg and it is removed by adding $Ca(OH)_2$ to hard water and precipitating these soluble bicarbonates in the form of insoluble salts.

$$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow CaCO_3\downarrow + 2H_2O$$
$$Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + Mg(OH)_2\downarrow + 2H_2O$$

18.

(d) 2

Explanation:

XeF_6 on complete hydrolysis produces XeO_3.

XeO_3 on reaction with OH^- produces $HXeO_4^-$ which on further treatment with OH^- undergo slow disproportionation reaction and produces XeO_6^{4-} along with Xe(g), $H_2O(l)$ and $O_2(g)$ as a by-product.

Oxidation half-cell in basic aqueous solution

$$HXeO_4^- + 5OH^- \rightarrow XeO_6^4 + 3H_2O + 2e^-$$

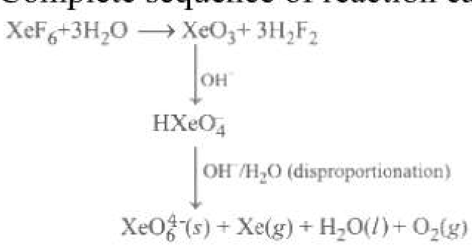
Reduction half-cell in basic aqueous solution

$$HXeO_4^- + 3H_2O + 6e^- \longrightarrow Xe + 7OH^-$$

Balanced overall disproportionation reaction is

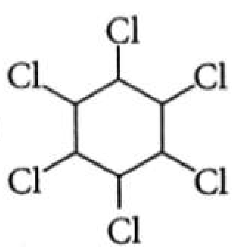
$$4HXeO_4^- + 8OH^- \rightarrow \underbrace{O3XeO_6^4 + Xe}_{\text{2 products}} + 6H_2O$$

Complete sequence of reaction can be shown as

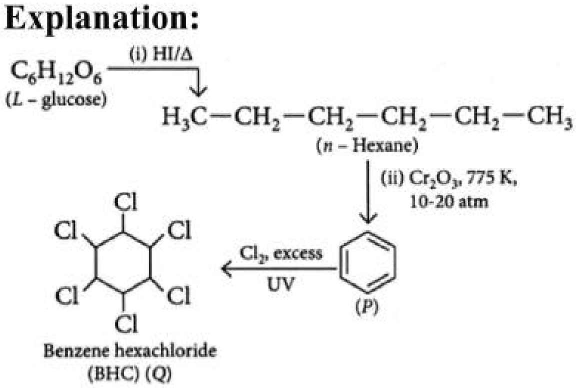


19.

(b)

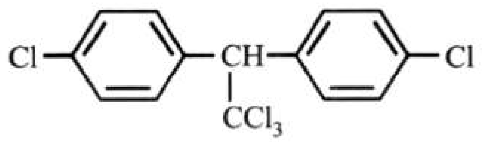


Explanation:



20.

(c)



Explanation:

Chloral on reaction with chlorobenzene in the presence of a catalytic amount of sulphuric acid forms DDT (dichloro diphenyl trichloroethane).

21. **(a)** H_2O is one of the products

(c) Stoichiometric coefficient of HSO_4^- is 6

(d) Iodide is oxidized

Explanation: Balancing the chemical equation by half-reaction method.

$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$

$2I^- \longrightarrow I_2 + 2e^- \times 2e^- \times 3$

$$\frac{ClO_3^- + 6H^+ + 6e^- \longrightarrow Cl^- + 3H_2O}{ClO_3^- + 6I^- + 6H^+ \longrightarrow 3I_2 + Cl^- + 3H_2O}$$

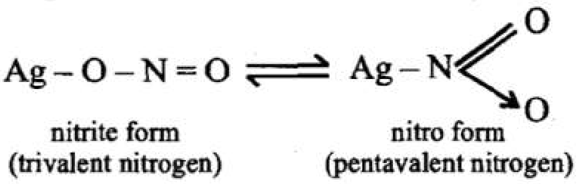
Adding $6HSO_4^-$ to both sides.

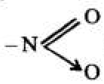
$ClO_3^- + 6I^- + 6H_2SO_4 \longrightarrow 3I_2 + Cl^- + 6HSO_4^- + 3H_2O$

22. **(c)** nitroethane

(d) ethyl nitrite

Explanation: Silver nitrite, being a salt of nitrous acid, occurs in two tautomeric forms.



NO_2^- ion from $AgNO_2$ may exist in two tautomeric forms, -O - N = O (nitrite ion) forming **alkyl nitrites,** and  nitro group) forming **nitroalkanes.**

$2C_2H_5Br + 2AgNO_2 \rightarrow C_2H_5NO + C_2H_5ONO + 2AgBr$

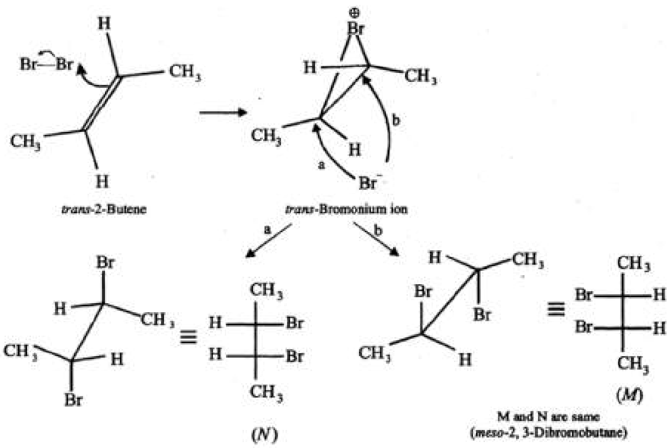
23. **(a)** (M and O) and (N and P) are two pairs of diastereomers

(c) Bromination proceeds through trans-addition in both the reactions

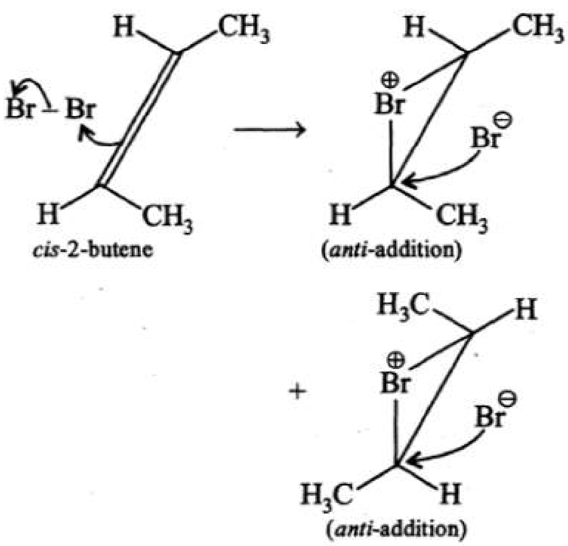
Explanation: M and N are identical. Further, O and P are enantiomers. Hence, M and O and N and P are two sets of diastereomers.

Bromination proceeds through trans- or anti-addition on alkenes, i.e., in both the reactions. Thus upon addition of bromine, trans-alkenes give meso-product, while cis-alkenes give enantomeric pair.

i.



ii.



24. 9

Explanation:

By observing the values of ionization enthalpy for atomic number (n + 2), it is observed that $I_2 >> I_1$. This shows that a number of valence shell electrons is 1 for atomic number (n + 2). Therefore element with an atomic number (n + 2) should be an alkali metal.

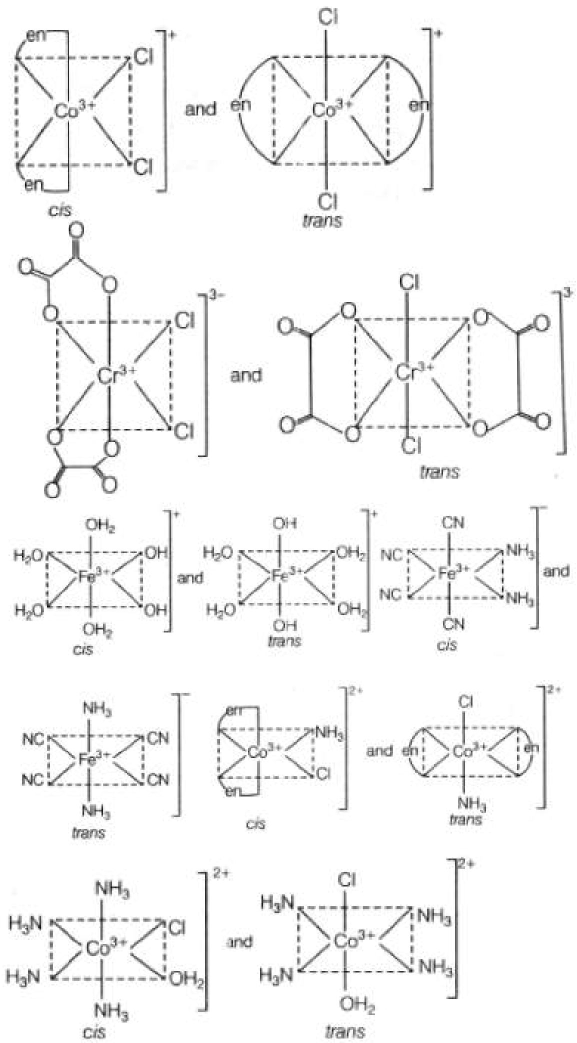
For atomic number (n + 3), $I_3 >> I_2$, which shows that it will be an alkaline earth metal.

All the observations suggest that atomic number (n + 1) should be a noble gas and atomic number (n) should belong to the halogen family. Since n < 10; hence n = 9.

25. 6

Explanation:

All six complex will show cis-trans isomerism.



26. 2500

Explanation:

Moles of acetic acid remains unadsorbed

$=$ Moles of $NaOH$ solution required $= 0.040\ L \times 1M$

$= 0.040\ mol$

Moles of acetic acid initially present $= 0.1\ L \times 0.5M$

$= 0.05\ mol$

Moles of acetic acid adsorbed $= 0.05 - 0.04 = 0.01\ mol$

Molecules of acetic acid adsorbed $= 0.01 \times N_A$

$0.01 N_A$ molecules occupy $= 1.5 \times 10^2\ m^2\ g^{-1}$ surface area

1 molecule will occupy $= \frac{1.5 \times 10^2}{0.01 \times 6.0 \times 10^{23}}$

$= 0.25 \times 10^{-19}\ m^2 = 2500 \times 10^{-23}\ m^2$

27. 27419

Explanation:

The shortest wavelength transition in the Balmer series corresponds to the transition

$n = 2 \rightarrow n = \infty$. Hence, $n_1 = 2$, $n_2 = \infty$

$\bar{\upsilon} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (109677\ \text{cm}^{-1})\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$= 27419.25\ \text{cm}^{-1}$

28. 6

Explanation:

$2MnO_2 + 4KOH + O_2 \longrightarrow 2K_2MnO_4 + 2H_2O$

Oxidation number of Mn in K_2MnO_4 is 6

K_2MnO_4; 2 + x - 8 = 0

x = 6

29. 24.14

Explanation:

$r_1 = kc_1$ and $r_2 = kc_2$

Since, rate of first-order reaction is directly proportional to the concentration of its reactant,

$\therefore \frac{r_1}{r_2} = \frac{c_1}{c_2} = \frac{0.04}{0.03}$

According to first-order reaction

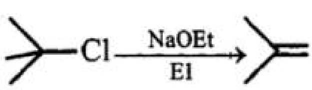
$k = \frac{2.303}{t_{20} - t_{10}} \log \frac{c_1}{c_2}$

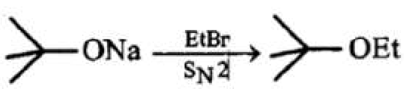
On substituting the various values $k = 0.0287\ \text{min}^{-1}$

$t_{\frac{1}{2}} = \frac{0.693}{k} = \frac{0.693}{0.0287} = 24.14$ min

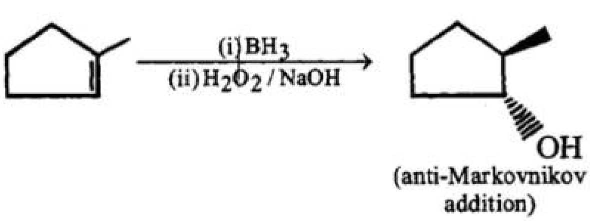
30. **(a)** (P) - (2), (Q) - (3), (R)- (1), (S) - (4)

Explanation:

(P) 

(Q) 

(R) 

(S) 

31.

(d) A - III, B - I, C - II, D - IV

Explanation:

HVZ reactions = Br_2/red P

Iodoform reaction = NaOH + I_2

Etard reaction = (i) $CrO_2\ Cl_2$, CS_2(ii) H_2O

Gattermann-Koch Reaction= CO, HCI, Anhydrous, $AICI_3$

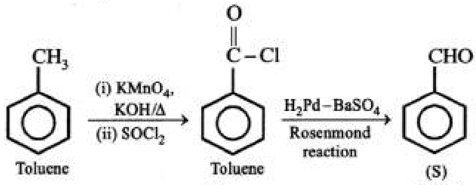
32.

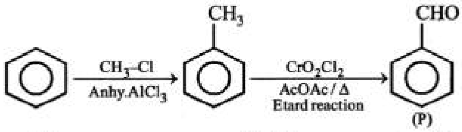
(d) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 5$; $S \rightarrow 2$

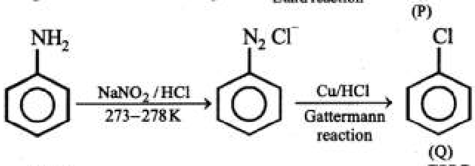
Explanation:

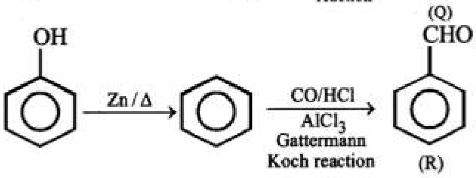
$P \rightarrow 3, Q \rightarrow 4, R \rightarrow 5, S \rightarrow 2$

$$\text{i. } \underset{\text{Acetophenone}}{Ph - \overset{O}{\overset{||}{C}} - CH_3} \xrightarrow{Zn-Hg/HCI} Ph - CH_2 - CH_3$$

ii. 

iii. 

iv. 

v. 

Maths

33.

(c) 3

Explanation:

$n! + (n+1)! + (n+2)! = n!\{1 + n + 1 + (n+2)(n+1)\} = n!(n+2)^2$

$\Rightarrow$ Either 7 divides $n+2$ or 49 divides $n!$

$\Rightarrow n = 5, 12, 14.$

34. **(a)** PX = -X

Explanation:

$P^T = 2P + I$

$\Rightarrow P = 2P^T + I \Rightarrow P = 2(2P + I) + I$

$\Rightarrow P = 4P + 3I \Rightarrow P + I = 0$

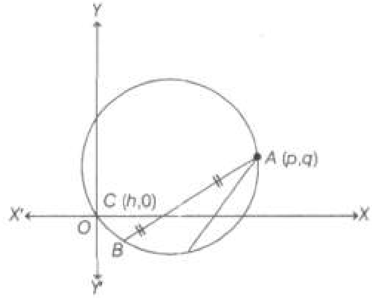
$\Rightarrow PX + X = 0 \Rightarrow PX = -X$

35.

(c) $p^2 > 8q^2$

Explanation:

From equation of circle it is clear that circle passes through origin. Let AB is chord of the circle.



$A \equiv (p, q) \cdot$ C is mid-point and coordinate of C is (h, 0)

Then, coordinates of B are (- p + 2h, - q) and B lies on the circle $x^2 + y^2 = px + qy$, we have

$(-p + 2h)^2 + (-q)^2 = p(-p + 2h) + q(-q)$

$\Rightarrow p^2 + 4h^2 - 4ph + q^2 = -p^2 + 2ph - q^{22}$

$\Rightarrow 2p^2 + 2q^2 - 6ph + 4h^2 = 0$

$\Rightarrow 2h^2 - 3ph + p^2 + q^2 = 0$ ...(i)

There are given two distinct chords which are bisected at X-axis then, there will be two distinct values of h satisfying Eq. (i).

So, discriminant of this quadratic equation must be > 0

$\Rightarrow D > 0$

$\Rightarrow (-3p)^2 - 4 \cdot 2 (p^2 + q^2) > 0$

$\Rightarrow 9p^2 - 8p^2 - 8q^2 > 0$

$\Rightarrow p^2 - 8q^2 > 0 \Rightarrow p^2 > 8q^2$

36. **(a)** {0, -1}

Explanation:

$f(x) = f^{-1}(x) \Rightarrow fof(x) = x$

$\Rightarrow [(x + 1)^2 - 1 + 1]^2 - 1 = x \Rightarrow (x + 1)^4 = x + 1$

$\Rightarrow (x + 1)[(x + 1)^3 - 1] = 0$

$\therefore x = 0$ or -1

$\therefore$ Required set is {0, -1}

37. **(b)** g(x) is continuous but not differentiable at b

(d) g(x) is continuous but not differentiable at a

Explanation: Clearly g(x) may or may not be continuous at x = a or x = b.

But it is continuous at all value of x except x = a, b.

Let us check the continuity of g(x) at x = a and x = b

$\lim_{x \to a^-} g(x) = 0$

$\lim_{x\to a^+} g(x) = \lim_{x\to a^+} \int_a^x f(t)dt = \int_a^a f(t)dt = 0$

and g(a)= $\int_a^a$ f(t) dt = 0

$\therefore$ g(x) is continuous at x = a

Also $\lim_{x\to b^-} g(x) = \lim_{x\to b^-} \int_a^x f(t)dt = \int_a^b f(t)dt$

and $\lim_{x\to b^+} g(x) = \int_a^b f(t)dt = \lim_{x\to b^-} g(x) = g(b)$

$\therefore$ g(x) is continuous at x = b

Therefore, g(x) is continuous $\forall x \in$ R

Now $g'(x) = \begin{cases} 0, & x < a \\ f(x), & a \leq x \leq b \\ 0, & x > b \end{cases}$

$g'(a^-) = 0$ and $g'(a_1^+) = f(a)$

$g'(b^-) = f(b)$ and $g'(b^+) = 0$

Since, $f(a), f(b) \in [1, \infty) \therefore f(a), f(b) \neq 0$

$\therefore g'(a^-) \neq g'(a^+)$ and $g'(b^-) \neq g'(b^+)$

$\Rightarrow$ g is not differentiable at a and b.

38. **(a)** 1

(d) 4

Explanation: Given that L_1 and L_2 are coplanar, therefore

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (5 - \alpha)[6 - 5\alpha + \alpha^2 - 2] = 0$

$\Rightarrow (5 - \alpha)(\alpha - 1)(\alpha - 4) = 0 \Rightarrow \alpha = 1, 4, 5.$

39. **(a)** the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for a > 0, b $\neq$ 0

(b) the y-axis for a = 0, b $\neq$ 0

(c) the x-axis for a $\neq$ 0, b = 0

Explanation: $z = \frac{1}{a+ibt} = x + iy$

$\Rightarrow x + iy = \frac{a-ibt}{a^2+b^2t^2} \Rightarrow x = \frac{a}{a^2+b^2t^2}, y = \frac{-bt}{a^2+b^2t^2}$

$\Rightarrow x^2 + y^2 = \frac{1}{a^2+b^2t^2} = \frac{x}{a} \Rightarrow x^2 + y^2 - \frac{x}{a} = 0$

$\therefore$ Locus of z a circle with centre $\left(\frac{1}{2a}, 0\right)$ and radius $\frac{1}{2|a|}$ irrespective of 'a' +ve or -ve

Also for b = 0, a $\neq$ 0, we get, y = 0

$\therefore$ Locus is x-axis

and for a = 0, b $\neq$ 0 we get x = 0

∴ locus is y-axis.

Hence, these are the correct options.

40. 7

Explanation:

Given $f(x) = 2x^3 - 15x^2 + 36x - 48$

and $A = \{x \mid x^2 + 20 \leq 9x\}$

$\Rightarrow A = \{x \mid x^2 - 9x + 20 \leq 0\}$

$\Rightarrow A = \{x \mid (x - 4)(x - 5) \leq 0\} \Rightarrow A = [4, 5]$

Also $f'\{x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6 (x - 2)(x - 3)$

Clearly $\forall$ x $\in$ A, f'(x) > 0

∴ f is strictly increasing function on A.

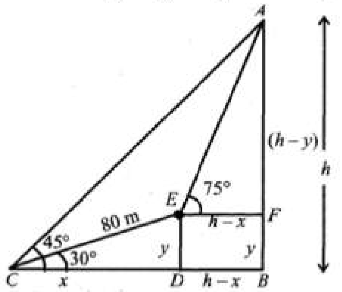
∴ Maximum value of f on A = f(5)

$= 2 \times 5^3 - 15 \times 5^2 + 36 \times 5 - 48 = 7$

41. 80.0

Explanation:

Let height (AB) = h m, CDA = x m and = y m



In rt. $\triangle CDE$,

$\sin 30^\circ = \frac{y}{80} \Rightarrow y = 40$

$\cos 30^\circ = \frac{x}{80} \Rightarrow x = 40\sqrt{3}$

Now, in $\triangle AEF$,

$\tan 75^\circ = \frac{h-y}{h-x} \Rightarrow (2+\sqrt{3}) = \frac{h-40}{h-40\sqrt{3}}$

$\Rightarrow (2+\sqrt{3})(h - 40\sqrt{3}) = h - 40$

$\Rightarrow 2h - 80\sqrt{3} + \sqrt{3}h - 120 = h - 40$

$\Rightarrow h + \sqrt{3}h = 80 + 80\sqrt{3} \Rightarrow (\sqrt{3}+1)h = 80(\sqrt{3}+1)$

$\therefore h = 80$ m

42. 9.0

Explanation:

Let locus point P(x, y).

∴ According to equation,

$\left|\frac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}x-y+1}{\sqrt{3}}\right| = \lambda^2$

$\Rightarrow \left|\frac{2x^2-(y-1)^2}{3}\right| = \lambda^2$

So, C : $|2x^2 - (y - 1)2| = \lambda^2$

Let the line y = 2x + 1 meets C at two points R (x_1, y_1) and $S(x_2, y_2)$

$\Rightarrow y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1$ ...(i)

$\Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

$\therefore RS = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$RS = \sqrt{5(x_1 - x_2)^2} = \sqrt{5}\,|x_1 - x_2|$

On solving equations curve C and line y = 2x + 1, we get

$|2x^2 - (2x)^2| = 3\lambda^2 \Rightarrow x^2 = \frac{3\lambda^2}{2}$

$\therefore RS = \sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right| = \sqrt{30}\lambda = \sqrt{270} \Rightarrow 30\lambda^2 = 270 \Rightarrow \lambda^2 = 9$

43. 24.5

Explanation:

Since p_2 = probability that minimum of chosen numbers is at most 40

= 1 - probability that minimum of chosen numbers is greater than or equal to 41

$= 1 - \frac{60\times60\times60}{100\times100\times100} = 1 - \frac{27}{125} = \frac{98}{125}$

$\therefore \frac{125}{4}p_2 = \frac{125}{4} \times \frac{98}{125} = 24.50$

44. 5

Explanation:

$\because$ The trace of A is 3

Let A = $\begin{bmatrix} x & y \\ z & 3-x \end{bmatrix}$

Now $A^2 = \begin{bmatrix} x & y \\ z & 3-x \end{bmatrix}\begin{bmatrix} x & y \\ z & 3-x \end{bmatrix}$

$= \begin{bmatrix} x^2+yz & xy+3y-xy \\ xz+3z-xz & yz+9+x^2-6x \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} x^2+yz & 3y \\ 3z & yz+9+x^2-6x \end{bmatrix}\begin{bmatrix} x & y \\ z & 3-x \end{bmatrix}$

$= \begin{bmatrix} x^3+xyz+3yz & x^2y+y^2z+9y-3xy \\ 3xz+yz^2+9z+x^2z-6xz & 6yz+27+3x^2-18x-xyz-9x-x^3+6x^2 \end{bmatrix}$

Given that trace of A^3 is -18

$\therefore x^3 + xyz + 3yz + 3yz + 27 + 3x^2 - 18x - xyz - 9x - x^3 + 6x^2$

= -18

$\Rightarrow 9yz + 9x^2 - 27x + 27 = -18 \Rightarrow yz + x^2 - 3x + 3 = -2$

$\Rightarrow 3x - x^2 - yz = 5$ ...(i)

Now, $|A| = 3x - x^2 - yz$

$|A| = 5$ From (i)

45. 2

Explanation:

Given: $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$

therefore we should have gof (x) = x

$\therefore g(f(x)) = x \Rightarrow g(x^3 + e^{\frac{x}{2}}) = x$

On differentiating both sides w.r.t. x, we get,

$g'(x^3 + e^{\frac{x}{2}}) \left(3x^2 + e^{\frac{x}{2}} \cdot \frac{1}{2}\right) = 1$

$\Rightarrow g'(x^3 + e^{\frac{x}{2}}) = \frac{1}{3x^2 + e^{\frac{x}{2}} \cdot \frac{1}{2}}$

For x = 0, we get g'(1) = 2

46.

(d) P → 4; Q → 6; R → 5; S → 2

Explanation:

Given 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

a. $\alpha_1 \rightarrow$ Total number of ways of selecting 3 boys and 2 girls from 6 boys and 5 girls

i.e., ${}^6C_3 \times {}^5C_2 = 20 \times 10 = 200 \therefore \alpha_1 = 200$

b. $\alpha_2 \rightarrow$ Total number of ways selecting at least 2 member and having equal number of boys and girls

i.e., ${}^6C_1{}^5C_1 + {}^6C_2{}^5C_2 + {}^6C_3{}^5C_3 + {}^6C_4{}^5C_4 + {}^6C_5{}^5C_5$

$= 30 + 150 + 200 + 75 + 6 = 461 \Rightarrow \alpha_2 = 461$

c. $\alpha_3 \rightarrow$ Total number of ways of selecting 5 members in which at least 2 of then girls

i.e., ${}^5C_2{}^6C_3 + {}^5C_3{}^6C_2 + {}^5C_4{}^6C_1 + {}^5C_5{}^6C_0$

$= 200 + 150 + 30 + 1 = 381 \Rightarrow \alpha_3 = 381$

d. $\alpha_4 \rightarrow$ Total number of ways for selecting 4 members in which at least two girls such that M_1 and G_1 are not included together.

G_1 is included $\rightarrow {}^4C_1 \cdot {}^5C_2 + {}^4C_2 \cdot {}^5C_1 + {}^4C_3$

$= 40 + 30 + 4 = 74$

M_1 is included $\rightarrow {}^4C_2 \cdot {}^5C_1 + {}^4C_3 = 30 + 4 = 34$

G_1 and M_1 both are not included

${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2$

$1 + 20 + 60 = 81$

$\therefore$ Total number = 74 + 34 + 81 = 189

$\alpha_4 = 189$

Now, P → 4; Q → 6; R → 5; S → 2

47. **(a)** (P) → (3), (Q) → (2), (R) → (4), (S) → (5)

Explanation:

Given

x_i	3	4	5	8	10	11
f_i	5	4	4	2	2	3

x_i	f_i	$x_i f_i$	C.F.	x_i - Mean	$f_i\|x_i$ - Mean$\|$	$\|x_i$ - Median$\|$	$f_i\|x_i$ - Median$\|$
3	5	15	5	3	15	2	10
4	4	16	9	2	8	1	4
5	4	20	13	1	4	0	0
8	2	16	15	2	4	3	6
10	2	20	17	4	8	5	10
11	3	33	20	5	15	6	18
	$\Sigma f_i =$ 20	$\Sigma x_i f_i =$ 120			$\Sigma f_i \mid x_i -$ Mean $\mid =$ 54		$\Sigma f_i \mid x_i -$ Median $\mid =$ 48

P. Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{20} = 6$

Q. Median $= \left(\frac{N}{2}\right)^{th}$ obs. $\left(\frac{20}{2}\right)^{th}$ obs. 10^{th} obs. = 5

R. Mean deviation about mean

$= \frac{\Sigma f_i|x_i - \text{Mean}|}{\Sigma f_i} = \frac{54}{20} = 2.70$

S. Mean deviation about median

$= \frac{\Sigma f_i|x_i - \text{Median}|}{\Sigma f_i} = \frac{48}{20} = 2.40$

48.

(c) P $\rightarrow$ 4, Q $\rightarrow$ 3, R $\rightarrow$ 2, S $\rightarrow$ 1

Explanation:

$P \rightarrow 4 : y = \cos\left(3\cos^{-1} x\right)$

$y = \cos\left[\cos^{-1}\left(4x^3 - 3x\right)\right]$

$y = 4x^3 - 3x$

$\Rightarrow \frac{dy}{dx} = 12x^2 - 3$ and $\frac{d^2y}{dx^2} = 24x$

$\therefore \frac{1}{y}\left\{\left(x^2 - 1\right)\frac{d^2y}{dx} + x\frac{dy}{dx}\right\}$

$= \frac{1}{4x^3 - 3x}\left\{\left(x^2 - 1\right)24x + x\left(12x^2 - 3\right)\right\}$

$= \frac{1}{4x^3 - 3x}\left\{36x^3 - 27x\right\}$

$= \frac{9\left\{4x^3 - 3x\right\}}{4x^3 - 3x} = 9$

$Q \rightarrow 3 : \because \vec{a_1}, \vec{a_2}, \ldots \vec{a_n}$ are position vectors of vertices $A_1, A_2, A_3, \ldots A_n$ of a regular polygon of n sides with its centre at origin.

$\therefore \left|\vec{a_1}\right| = \left|\vec{a_2}\right| = \ldots . = \left|\vec{a_n}\right| = \lambda$

Now, $\overrightarrow{a_k} \times \overrightarrow{a_{k+1}} = \lambda^2 \sin \frac{2\pi}{n} \hat{n}$

and $\overrightarrow{a_k} \cdot \overrightarrow{a_{k+1}} = \lambda^2 \cos \frac{2\pi}{n}$

$\therefore \left| \sum_{k=1}^{n-1} (\vec{a}_k \times \vec{a}_{k+1}) \right|$

$= \left| \sum_{k=1}^{n-1} (\vec{a}_k \cdot \vec{a}_{k+1}) \right|$

$\Rightarrow (n-1)\lambda^2 \sin \frac{2\pi}{n} = (n-1)\lambda^2 \cos \frac{2\pi}{n}$

$\Rightarrow \tan \frac{2\pi}{n} = 1 \Rightarrow \frac{2\pi}{n} = \frac{\pi}{4} \Rightarrow n = 8$

R $\rightarrow$ 2 : Normal from P(h, 1) on $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is $\frac{x-h}{\frac{h}{6}} = \frac{y-1}{\frac{1}{3}}$

$\Rightarrow$ 2(x - h) = h(y - 1)

$\Rightarrow$ 2x - hy - h = 0

Slope of Normal $= \frac{2}{h}$

It is perpendicular to x + y = 8

$\therefore \frac{2}{h}x - 1 = -1 \Rightarrow h = 2$

$S \rightarrow 1 : \tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{2x+1} \cdot \frac{1}{4x+1}}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{6x+2}{8x^2+6x}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \frac{3x+1}{4x^2+3x} = \frac{2}{x^2} \Rightarrow 3x^2 - 7x - 6 = 0$

$\Rightarrow$ x = 3 or $-\frac{2}{3}$

Since $x > 0$

$\therefore$ Only one +ve solution is there

Hence P $\rightarrow$ 4, Q $\rightarrow$ 3, R $\rightarrow$ 2, S $\rightarrow$ 1 is the correct option.